An inverse method for material parameters determination of titanium samples under tensile loading

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ABSTRACT: The present study focus on the parameters identification of an elastic-plastic behaviour law by the mean of full-field measurements and inverse method. Titanium samples are loaded under tensile conditions. Measurements provides displacement fields that are used to update a Finite Element (FE) model. The minimization of the cost-function is ensured by a Levenberg-Marquardt (LM) algorithm. The whole deformation process is considered even while necking occurs. Finally, five parameters are successfully identified. The resulting force assessed by the FE model with the identified parameters point the mismatching of the Hollomon's law when necking rules the deformation.

KEYWORDS: Full-field measurement, Inverse method, Tensile test, FE simulation, Titanium.

1 INTRODUCTION

In the recent years, several studies has been led to develop the use of inverse method in parameters identification. Uniaxial tensile test on standard samples is broadly use for models validation. In those studies, tensile test is seldom considered until the end of the necking process [1]. This study focus on a parameters identification that take into account the whole deformation process: including high strain necking.

2 FULL-FIELD MEASUREMENTS

2.1 Experimental setup

Uniaxial tensile tests are carried out on Commercially Pure (CP) titanium samples. Those are clamped into a $5\ kN$ Instron tensile machine and digital images are recorded during the deformation process with a $1\ Hz$ frequency until the sample's ruin. The sample's surface is previously speckled with black and white painting sprays in order to allow Digital Image Correlation (DIC). The gauged section size initially equals 35×10 mm^2 and the sheet thickness is 0.5 mm.

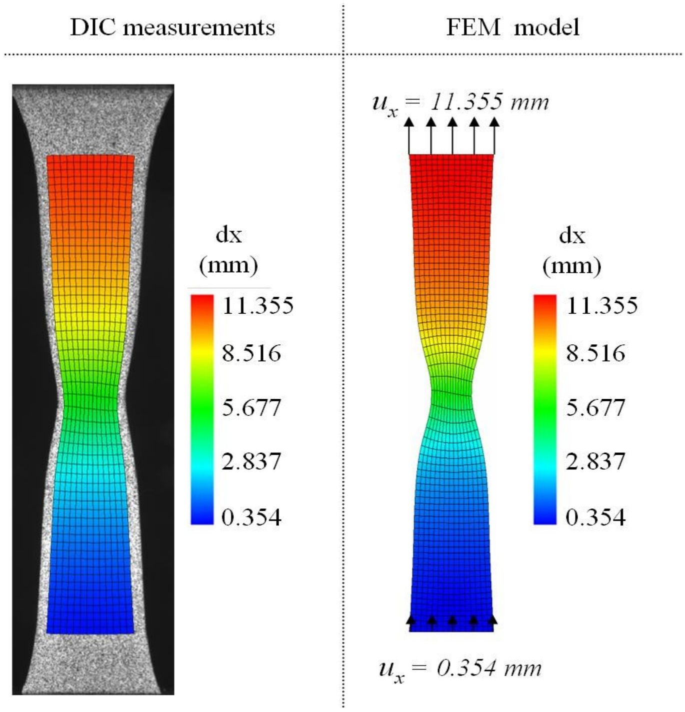

Figure 1: Measured displacement field upon the sample picture just before failure. FE model that repeat the measured boundaries conditions.

2.2 Digital Image Correlation

Once the test has been led, the video data can be compute to provide both longitudinal and transverse displacement fields (respectively u_x and u_y). The DIC method is performed using 7D software developed by Vacher [2]. For this test, the grid of analysis consists in 55×13 quadrilaterals of 12 by 12 pixels, which corresponds to an area of analysis of approximately 285 mm^2. The extensometric base is a square of 12 by 12 pixels. The estimated error in necking area is about 0.1 pixels corresponding to $5.26\ \mu$m.

2.3 Material constitutive equations

In this study, CP titanium is assumed to exhibit an anisotropic elastic-plastic behaviour. The Young's modulus is set to $E = 111800$ MPa and the Poisson's ratio to $\nu = 0.34$. The hardening curve is defined by a Hollomon's law as following:

$$\sigma = \sigma_y + K\epsilon_p^n \tag{1}$$

where $\sigma_y = 368$ MPa is the yield stress, ϵ_p is the plastic strain, K and n are two parameters to be identified. The anisotropic behaviour is represented by the Hill criterion:

$$\begin{aligned} f(\underline{\sigma}) &= \Big(\frac{1}{2}\Big(F(\sigma_{22}-\sigma_{33})^2 + G(\sigma_{33}-\sigma_{11})^2 + \\ &\quad + H(\sigma_{11}-\sigma_{22})^2 + 2L\sigma_{23}^2 + 2M\sigma_{31}^2 + \\ &\quad + 2N\sigma_{12}^2\Big)\Big)^{\frac{1}{2}} \end{aligned} \tag{2}$$

where F, G, H, L, M, N are material parameters characterizing the anisotropy. In the case of a sheet-formed material with the plane stress assumption and if the yield stress σ_y at 0° is taken as reference, this criterion amounts to an expression showing only 3 parameters as defined in Abaqus by the following yield stress ratio:

$$R_{22} = \sqrt{\frac{2}{F+H}}\ ; R_{33} = \sqrt{\frac{2}{F+G}}\ ; R_{12} = \sqrt{\frac{2}{3}N} \tag{3}$$

3 IDENTIFICATION SCHEME

3.1 The inverse method

The inverse method approach propose to extract constitutive parameters from the measurements of heterogeneous strain fields. Table 1 summarizes knowns and unknowns of both direct and inverse methods [3].

Table 1: Direct and inverse problems

	Direct	Inverse problem
known	Geometry	Geometry
	Loading distribution f	Resulting force F
	Constitutive equations	Constitutive equations
	Constitutive parameters	u or ϵ
unknown	u, ϵ, σ	Constitutive parameters

Geometry, resulting force and the constitutive equations are set *a priori* then the only materials parameters ($K, n, R_{22}, R_{33}, R_{12}$) will be identified in this study. In 1991, Hendricks [4] first proposed a solution of this latter problem based on the Finite Element Method (FEM). It consist in updating of a finite element model in order to assess numerically displacement fields that can be compared to the experimental ones. This resolution method can be represented by the following block diagram (figure 2).

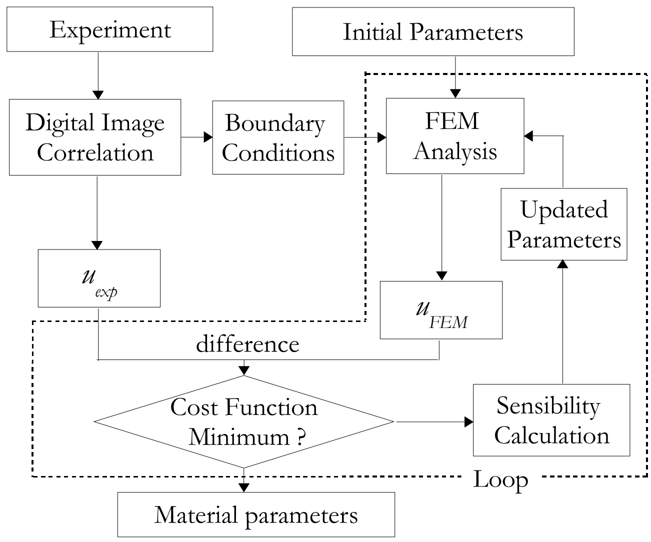

Figure 2: Block diagram of the parameters identification problem.

3.2 Optimization loop

The identification of the five parameters is based on a LM optimization method. The cost function to be minimized is expressed by the least-square difference of measured and calculated displacement fields such as:

$$\begin{aligned} C(\underline{p}) &= \sum_{i=1}^{m}\sum_{j=1}^{n}\Big((u_{x_{ij}}^{FEM}(\underline{p}) - u_{x_{ij}}^{exp})^2 + \\ &\quad + (u_{y_{ij}}^{FEM}(\underline{p}) - u_{y_{ij}}^{exp})^2\Big)^{1/2} \end{aligned} \tag{4}$$

where $\underline{p}$ is the unknown parameters column, n the number of measured points and m the number of step times taken into account. As introduced by Lecompte [5], the cost function is neither weighed [4, 6] nor using penalty terms [7]. It is expressed only by the mean of displacements. The minimum of the cost function is found by solving the iterative scheme presented in equation (5).

$$\left[\underline{\underline{J}}^T\underline{\underline{J}} + \lambda_{LM}\underline{\underline{I}}\right]\left(\frac{\underline{p}_{k+1} - \underline{p}_k}{\underline{p}_k}\right) = \underline{\underline{J}}^T(\underline{u}^{FEM}(\underline{p}_k) - \underline{u}^{exp}) \tag{5}$$

where $\underline{\underline{I}}$ is the identity matrix, λ_{LM} is the Levenberg-Marquardt multiplier such as defined by Kleinermann in [8] and $\underline{\underline{J}}$ is the sensitivity matrix. This latter matrix is compute using the following forward finite difference scheme:

$$J_{ij} = \frac{\partial \underline{u}_i^{FEM}(\underline{p})}{\partial \underline{p}_j} \approx \frac{\underline{u}_i^{FEM}(\underline{p}, p_j + \Delta_j) - \underline{u}_i^{FEM}(\underline{p})}{\delta} \tag{6}$$

where Δ and δ are defined by $\Delta_j = \delta \cdot p_j$ and δ is arbitrary set to 0.002.

4 NUMERICAL MODEL

4.1 *Finite element analysis*

The experimental conditions has been repeated in Abaqus FE code. A shell model of the DIC grid has been created and meshed with quadrangular elements. The loading conditions that were applied to the both ends of this grid has been picked from the DIC analysis and match the measured experimental displacements of the DIC grid (see figure 1). For the sake of numerical stability the FE mesh has been refined to 70×15 elements. In order to compare numerical and experimental displacements at the same material points, interpolation was led on the numerical fields. It provides the numerical displacements values at the same points than the experimental ones. In order to ensure a reasonable calculation time, only 5 step times were considered. Those step times were uniformly spread along the hardening curve from the beginning to the end of the plastic zone. Since, necking is also considered and the last step time was chosen to match the last instant before sample's failure. As a consequence, both experiment and FEM analysis are led until the end of the necking phenomenon. When the sample reach this stage of deformation, it exhibits shear in the neck area, this allows the identification of the three anisotropic parameters define by the Hill's criterion (see section 2.3).

5 Results

The estimated value of the five parameters to identify and the problem's cost function are summarized in Table 2. The start point of the anisotropic parameters was chosen such as the initial behaviour match the isotropic case.

Table 2: Material parameters and cost function values

	initial values	final values
K	450	443.79
n	0.3	0.5054
R_{22}	1	1.054
R_{33}	1	1.3086
R_{12}	1	0.8823
$C(\underline{p})$	$19.4 \cdot 10^{-3}$	$9.4 \cdot 10^{-3}$
CPU (hrs)[1]	-	8.5

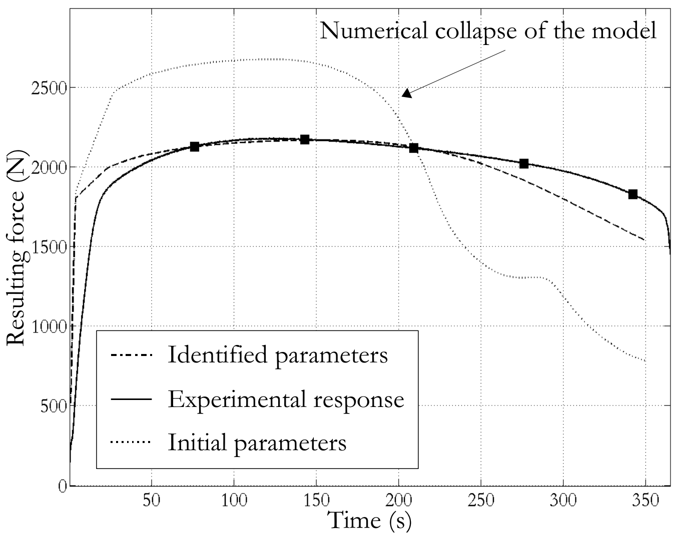

Figure 3: Calculated and measured resulting force. Markers represents the considered step times.

The measured resulting force F can be plotted versus the calculated one (figure 3). The dashed curves represents the numerical response assessed us-

[1]processor: 3 GHz, RAM: 512 Mo

ing respectively the identified and the initial parameters, while the solid curve represents the experimental response. The initial set of parameters is responsible of a numerical collapse of the model before the end of the prescribed displacement. Indeed this set induce a early failure of the sample as shown by figure 3. The improvement on model fitting can be verified by plotting the cost-function value versus the iteration number (figure 4).

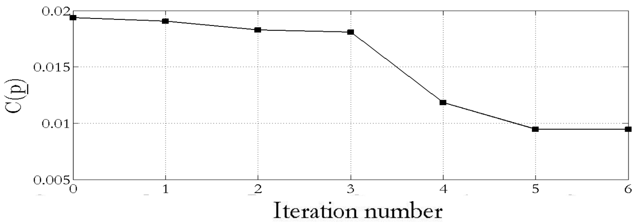

Figure 4: Decreasing of the cost-function value versus iteration number.

However, the fitting of the elastic behaviour is poor and the experimental Young's modulus value seems to be different than the chosen one. In order to improve the numerical response further studies will focused on the identification on elastic parameters. Moreover, it can be seen on figure 3 that the fitting of numerical model on experimental response is getting rougher when the necking phenomenon becomes significant. This is pointing the mismatching of the Hollomon law with the plastic behaviour at high strains and especially when necking rules the material deformation.

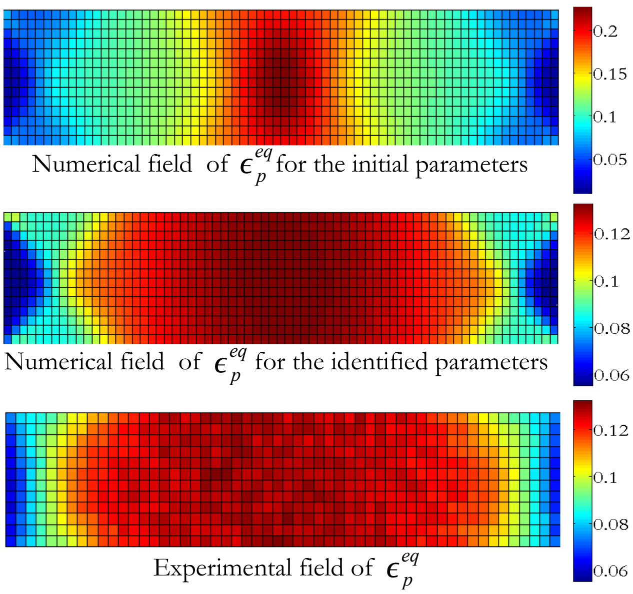

Figure 5: Equivalent plastic strain fields ϵ_p^{eq} for initial and identified models vs. the experimental field (step time 146 s).

Figure 5 presents equivalent plastic strain fields for both initial and identified parameters compared to the experimental measurement. A rather good agreement is obtained even if boundary conditions are somehow different.

6 CONCLUSIONS

A mixed numerical-experimental method has been used to identify material parameters under tensile loading conditions. This approach consists in minimizing a cost-function based on both experimental and numerical displacement fields. The presented results point the efficiency of the LM algorithm to provide accurate values (displacements and loading) after only 6 iterations. However, the accuracy of the presented method can be improved by performing tests leading to more heterogeneous fields (e.g. deep drawing forming, bulge test...).

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