The Facet Method for Plastic Anisotropy of Textured Materials

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ABSTRACT: The Facet method is a new approach to implement the plastic anisotropic behaviour of polycrystalline materials in finite-element models for simulating metal forming processes. It employs analytical expressions of plastic potentials in strain rate space and/or stress space. The parameters in these expressions are obtained by fitting to the predictions of a multilevel model for the plastic deformation of a textured material. The resulting equipotential surfaces in strain rate space and yield loci in stress space are automatically convex. Examples of *q*-values and uniaxial yield stress variations as obtained with the Facet method in combination with the Taylor theory are shown for three industrial sheet metals. The results are compared to the ones calculated directly from the Taylor theory and those obtained with the Quantic method.

Key words: aluminum, anisotropy, plasticity, finite element analysis, texture, steel

1 INTRODUCTION

Two quite different approaches exist to describe the plastic anisotropy of polycrystalline materials within finite element (FE) formulations for the simulation of forming processes. In the first, the FE program makes a call to a multilevel model each time the stress-strain relationship is needed at the engineering length scale, and the multilevel model provides the average response of a representative volume element of grains. In the other, the anisotropic stress-strain relations are described by analytical expressions such as yield locus formulations. The parameters in these expressions can be either obtained from real mechanical tests [1], or from virtual tests using a multilevel model to obtain the anisotropic response of the material. The latter approach is adopted in previous work of Van Houtte and Van Bael [2,3]. They have used the theory of plastic potentials in strain rate space and stress space [4-7] in combination with the Taylor theory [8] as multilevel model. In spite of successful FE implementations, the simulations in their earlier work [2] occasionally suffered from divergence problems caused by the non-convexity of the yield surface in some vertices. The problem of non-convexity was solved thanks to a new mathematical expression for the plastic potential in strain rate space [3], which is referred to as the "Quantic method". A mathematical criterion could be formulated to ensure the overall convexity of the yield surface, and an iterative algorithm was proposed to slightly modify the fitted parameters in order to satisfy that criterion. The Quantic method has been implemented in both implicit and explicit finite-element software programs through user material routines [9]. In spite of the guaranteed convexity and the resulting reliability of the method, the Quantic method has the drawback of being formulated in strain rate space but not in stress space. This makes it computationally demanding to verify whether plastic yielding occurs and to satisfy the yield criterion in both implicit and explicit FE formulations. The Facet method proposed in this paper overcomes this limitation, and can also be adapted for materials with stress differential effects [10]. First results of *q*-values and uniaxial yield stress variations obtained with the Facet method for an aluminium alloy and two steels are presented in this paper. Examples of equipotential surfaces and yield locus sections for the same materials can be found in [11] and [12], respectively.

2 THE FACET METHOD

2.1 Strain rate space

Van Houtte et al. [3,7,10] describe the plastic anisotropy of textured polycrystalline materials by means of a plastic potential $\psi(\mathbf{D})$ in strain rate space. The potential is identified as the plastic work dissipated per unit volume in function of the macroscopic plastic strain rate **D**. For rate-insensitive materials, the deviatoric stress tensor **S** derived from the plastic flow stress can be calculated for a given tensor **D** as:

$$\mathbf{S} = \frac{\partial \psi(\mathbf{D})}{\partial \mathbf{D}} \tag{1}$$

The following generic expression is employed for the function $\psi(\mathbf{D})$:

$$\psi(\mathbf{D}) = \left[G_n(\mathbf{D})\right]^{1/n} \tag{2}$$

$G_n(\mathbf{D})$ is a homogenous polynomial of degree n in terms of the components of the tensor **D**.

In the Quantic method the following series expansion is used for $G_n(\mathbf{D})$ (in case of n=6) [3]:

$$G_n(D) = \alpha'_{pqrstu} D_p D_q D_r D_s D_t D_u \quad \text{with} \quad 1 \le p \le q \le r \le s \le t \le u \le 5 \tag{3}$$

D_p (p=1..5) are the components of a 5-dimensional vector representation of **D**. The expression contains 210 parameters α'_{pqrstu}.

The Facet method [10] uses the following algebraic expression for the function $G_n(\mathbf{D})$:

$$G_n(\mathbf{D}) = \sum_{\kappa=1}^{K} \lambda_\kappa \left(s_{\kappa p} D_p\right)^n \quad \text{and} \quad \lambda_\kappa \ge 0 \tag{4}$$

λ_κ and $\mathbf{s}_\kappa$ (i.e. a tensor with components $s_{\kappa p}$) are the parameters, and the exponent n is an even natural number. In the present work, the value 6 is used for n. It can be shown that equipotential surfaces in strain rate space (i.e. surfaces that contain all **D** for which $\psi(\mathbf{D})$ is a constant) as obtained by equations (2) and (4) are always convex, and so are the corresponding yield loci [10]. In order to identify the parameters λ_κ and $\mathbf{s}_\kappa$ a multilevel model is applied first to obtain the parameters $\mathbf{s}_\kappa$ as the deviatoric yield stresses for K different directions in strain rate space. Here, 402 such directions are considered, defining an almost equidistant grid of points on a hypersphere in the 5-dimensional strain rate space. A very efficient fitting procedure is then applied to derive also the parameters λ_κ [10].

It is remarked that after elaboration of all terms in equation (4) the function $G_n(\mathbf{D})$ can be written in the same form as equation (3). The adopted expression in the Facet method in strain rate space thus is a particular case of the Quantic potential.

2.2 Stress space

A plastic potential also exists in stress space, which in case of rate-insensitivity is given by the following equations [7,10]:

$$G'_n(\mathbf{S}) = 1 \tag{5}$$

$$\mathbf{D} = \lambda \frac{\partial G'_n(\mathbf{S})}{\partial \mathbf{S}} \quad \text{with } \lambda \ge 0 \tag{6}$$

The surface in stress space described by equation (5) is known as the yield locus, and equation (6) represents the normality rule. $G'_n(\mathbf{S})$ is a homogenous polynomial of degree n in terms of the components of **S**. In the Facet method the following expression is proposed for $G'_n(\mathbf{S})$ [10]:

$$G'_n(\mathbf{S}) = \sum_{\kappa=1}^{K} \lambda'_\kappa \left(d_{\kappa p} S_p\right)^n \tag{7}$$

S_p (p=1..5) are the components of a 5-dimensional vector representation of a deviatoric stress tensor **S** that corresponds to a plastic flow stress if equation (5) is satisfied. λ'_κ and $\mathbf{d}_\kappa$ (i.e. a tensor with components $d_{\kappa p}$) are the parameters. The exponent n is an even natural number, and the value 6 is used here. By analogy to the Facet method in strain rate space, equations (5) and (7) automatically guarantee the convexity of the yield locus and the corresponding equipotential surfaces in strain rate space. Since the multilevel models are usually not stress driven, the procedure to identify the parameters λ'_κ and $\mathbf{d}_\kappa$ can not simply be mirrored from the one to identify the parameters λ_κ and $\mathbf{s}_\kappa$. The challenge is to find the parameters $\mathbf{d}_\kappa$ as plastic strain rates that correspond to yield stresses in K equidistant directions in stress space. Three strategies have been tested to obtain λ'_κ and $\mathbf{d}_\kappa$ [11]. The "second strategy" is used here, in view of its better reproduction of flat yield locus parts [11]. In this approach, the potential in strain rate space is derived first. The identification then proceeds by analogy to the procedure in strain rate space, but this time using $G_n(\mathbf{D})$ instead of the multilevel model to obtain the parameters $\mathbf{d}_\kappa$ as plastic strain rates that correspond to 402 nearly equidistant directions in stress space.

3 RESULTS

Three industrial sheet metals are considered here: an aluminium alloy and a low-carbon steel, both with textures of moderate intensity (T.I. of 2.31 and T.I. 1.85, resp.), and an interstitial free (IF) steel with a strong texture (T.I. 4.84). The Taylor theory [8] has been used, assuming identical critical resolved shear stresses on all slip systems, with {111}<110> slip for the aluminium, and {110}+{112}<111> slip for the steels. Figure 1 shows the *q*-value, i.e. the ratio of the plastic strain in the width direction to the one in the longitudinal direction) in function of the angle α of the tensile specimen to the rolling direction.

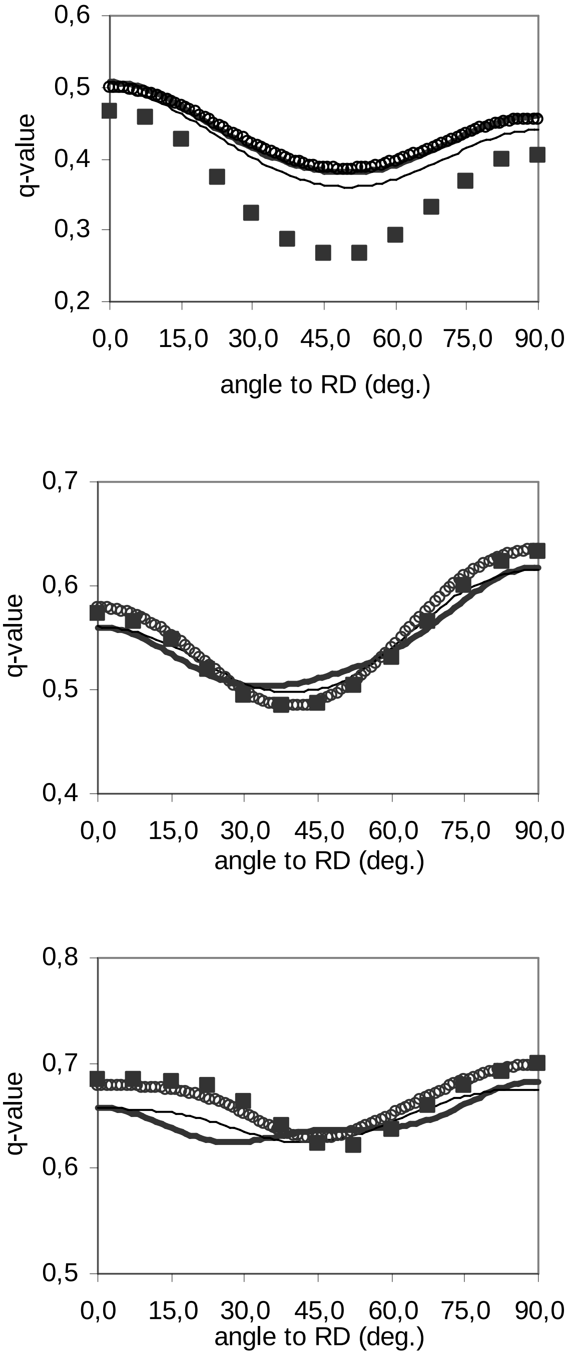

Fig. 1: *q*-values in function of the angle to the rolling direction for (a) Aluminium 6016-T4 alloy (T.I. 2.31); (b) Low carbon steel (T.I. 1.85); (c) IF steel (T.I. 4.84). Open dots: Quantic method. Thin lines: Facet method in strain rate space. Thick lines: Facet method in stress space. Filled squares: direct Taylor results.

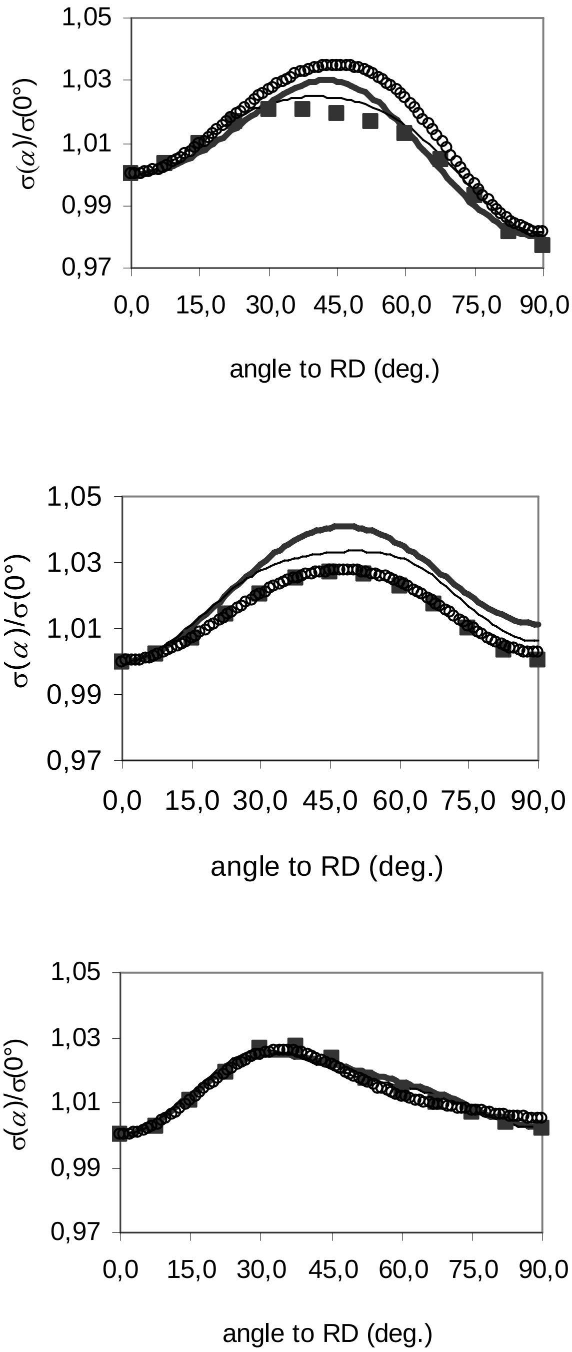

Fig. 2: Normalised yield stresses in function of the angle to the rolling direction for (a) Aluminium 6016-T4 alloy (T.I. 2.31); (b) Low carbon steel (T.I. 1.85); (c) IF steel (T.I. 4.84). Open dots: Quantic method. Thin lines: Facet method in strain rate space. Thick lines: Facet method in stress space. Filled squares: direct Taylor results.

Figure 2 gives the uniaxial yield stress normalised to the yield stress in the rolling direction, in function of the angle α. The results are obtained with the Facet method in strain rate space, the Facet method in stress space, the Quantic method in strain rate space, and direct Taylor calculations. The degree 6 is used for the Facet and Quantic methods.

4 DISCUSSION AND CONCLUSIONS

All results obtained with the Facet method in strain rate space only slightly deviate from those obtained with the Quantic method, inspite of the totally different parameter identification procedure. The overestimation of the q-values for the aluminium alloy 6016-T4 was already reported for the Quantic method in terms of r-values [13] ($q = r/(1+r)$), and this was found to be a result of the procedure to modify the parameters to guarantee convexity. The Facet method in stress space generates similarly shaped curves, except for the q-values of the IF steel with a strong texture where a spurious local maximum appears near 45°. Further research is required to find out whether the capability of the Facet methods to mimic the Taylor model can be improved by using higher degree expressions. This may also necessitate the use of a denser grid on the 5-dimensional unit spheres in strain and/or stress space for the parameter identification [10].

The quality of the Facet method in strain rate space is as good as that of the Quantic method, but the new method offers several advantages. Both theory and software implementations are much simpler. Also, the multilevel model needs to be performed for only 402 plastic strain rates (as opposed to ±100000), since the complete stress tensors for the considered strain rates are fully employed, and not only the rates of plastic work. The Facet method can therefore also be used in combination with more complex and computationally expensive multilevel models, such as a self-consistent model [14], the ALAMEL-model [15], or a crystal-plasticity finite element model (CPFEM) [16]. Finally, the Facet method in stress space is expected to allow for CPU savings during finite-element simulations [12].

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