Computational determination of the mechanical behavior of textile composite reinforcement. Validation with x-ray tomography.

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ABSTRACT: The knowledge of the mechanical behavior of woven fabrics is necessary in many applications in particular for the simulation of textile composite forming. This mechanical behavior is very specific because of the possible motions between the fibers and the yarns. The objective of this presentation is to introduce 3D mesoscopic finite element analyses of woven reinforcement shear aimed at determining their macroscopic mechanical behavior and local results such as the deformed shape of yarns. These types of results can be used for various applications such as simulations of composite forming or fluid flow simulations inside composite preforms. Since yarns are made of thousands of fibers it is not possible to model each of them and an equivalent continuum mechanics model is developed within the hypo-elastic theory. This model has to render the fibrous nature of the yarn, which requires using specific objective rates and material properties. From a macroscopic point of view, the shear behavior computed from simulations is compared to experiments, showing a good agreement.

Key words: Textile composites, Meso-macro analysis, Hypoelasticity, Fibrous material, In-plane shear.

1 INTRODUCTION

The RTM process for composite material forming consists of three stages. A dry textile reinforcement is formed (performing stage), then the resin is injected within this preform and cured to obtain the final composite part. During the first stage, the reinforcement undergoes in-plane deformations as biaxial tension, in-plane shear, compaction, bending. These deformations can be large especially in-plane shear which is essential in the case of double curved shapes. These macroscopic deformations are directly related to mesoscopic deformations of the reinforcement (at the scale of the yarns). For instance large in-plane shear of the reinforcement leads to a significant lateral crushing of the yarns. As a consequence, the macroscopic behaviour is related to its mesoscopic behaviour. Moreover the local deformations can modify the mechanical properties of the reinforcement and its permeability.
The objective of this paper is to present a method for the simulation of deformations of a woven composite reinforcement representative unit cell (i.e. at mesoscopic scale). These simulations enable to determine the macroscopic mechanical behaviour at large strain of dry reinforcements. This mechanical behaviour is necessary in finite element simulations of the performing stage. Besides, knowing the deformed geometry of the woven cell enables to determine the permeability of the fibrous reinforcement via Stokes (or Stokes Brinkman) flow simulations within this deformed cell. At last the geometry of the deformed reinforcement heavily influences the mechanical behaviour of the final composite part. In particular, meso-scale damage prediction simulations require knowing this geometry.
The yarn constitutive model used in the following analyses is based on a hypo-elastic approach. The behaviour of the yarn is very specific since it is made of thousands of fibers which can slide with respect to each other. Therefore the objective derivative used in the yarn hypo-elastic constitutive model has to be governed by the fiber direction. The

yarn is supposed to be transversely isotropic. The parameters of the material model are identified by an inverse method. In this paper, an example of in-plane shear of a unit cell is shown and compared with experimental results on the mechanical point of view.

2 CONSTITUTIVE EQUATION OF THE FIBROUS MATERIAL AT FINITE STRAIN

In a FE analysis of the deformation of a woven RUC, the yarn will be considered as a continuum because there are too many fibres; each of them can not be modelled separately. Nevertheless the mechanical behaviour model of this yarn has to account for its fibrous nature, especially the large difference of rigidity between the fibre direction and the other directions. This requires, in particular, to accurately follow the direction of fibres.

An orthotropic hypo-elastic constitutive equation (or orthotropic elastic rate constitutive equation) is used. Rate constitutive equations are widely used in simulation codes at finite strains [1]. These equations consider the stress rate with respect to a rotated frame fitting the matter at best. The Cauchy stress rate is related to the strain rate **D** by the elastic constitutive tensor **C**:

$$\underline{\underline{\sigma}}^{\nabla} = \underline{\underline{\underline{\underline{C}}}} : \underline{\underline{D}} \tag{1}$$

The objective derivative of the Cauchy stress **σ** is defined by the rotation of the rotated frame:

$$\underline{\underline{\sigma}}^{\nabla} = \underline{\underline{Q}}.\left(\frac{d}{dt}\left(\underline{\underline{Q}}^{T}.\underline{\underline{\sigma}}.\underline{\underline{Q}}\right)\right).\underline{\underline{Q}}^{T} \tag{2}$$

where **Q** is the rotation that defines the rotated frame. This leads, through cumulating in the rotated frame, to the following Cauchy stress tensor:

$$\underline{\underline{\sigma}} = \underline{\underline{Q}} \cdot \left(\int_0^t \underline{\underline{Q}}^{T} \cdot \left(\underline{\underline{\underline{\underline{C}}}} : \underline{\underline{D}}\right) \cdot \underline{\underline{Q}}\, dt\right) \cdot \underline{\underline{Q}}^{T} \tag{3}$$

The rotations **Q** that are classically used in simulation codes are the polar rotation (defining then Green Naghdi's objective derivative) or the rotation of the corotational frame (Jaumann's objective derivative). It was shown [2] that the constitutive laws based on these rotations can not correctly describe finite strains of a fibrous medium. These rotations are in fact averages of the rotations of the material directions at one point. In the case of fibrous media, it is needed to give preference to the fibre direction. A hypo-elastic constitutive law adapted for fibrous media like yarns is obtained when using the rotation of the fibre in equations (2)-(3) [2]. The current fibre direction $\mathbf{f}_1$ is known from the gradient tensor **F**:

$$\underline{f}_1 = \frac{\underline{\underline{F}}.\underline{f}_1^0}{\left\|\underline{\underline{F}}.\underline{f}_1^0\right\|} \tag{4}$$

where $\mathbf{f}_1^0$ is the initial fibre direction. The other base vectors are also obtained from **F** and projected in the plan transverse to $\mathbf{f}_1$ so as to build an orthonormal base. From the knowledge of the initial basis $\{\mathbf{f}^0\}$ and the current one $\{\mathbf{f}\}$, the rotation **Φ** of the so called fiber frame can be computed and used in (2) and (3):

$$\underline{\underline{\Phi}} = \underline{f}_i \otimes \underline{f}_i^0 \tag{5}$$

Using the rotation of the fibre in the objective derivative and the rotated frame brings to summing the stress increments precisely in the fibre frame. In this frame, the components along the fibre direction and the transverse ones can be distinguished.

The transverse behaviour in plane ($\mathbf{f}_2$; $\mathbf{f}_3$) is assumed to be isotropic (though unhomogeneous). This assumption is supported by high resolution tomography observations [3] made on deformed and undeformed reinforcements (fig. 1).

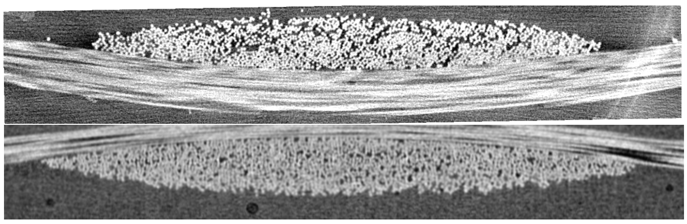

Fig. 1. Tomography reconstructed slices of a glass plain weave. Undeformed state (top) and biaxial tensioned state (bottom). (different scales)

The fibre direction modulus is obtained by a tensile test on a yarn. It is considered as constant. The transverse modulus is related to the longitudinal and transverse strains. If the yarn undergoes longitudinal tension, it becomes transversely much stiffer. It is also very little stiff transversally when transverse compression is low and it becomes stiffer as compression increases. The following form is used for the transverse modulus E_T:

$$E_T\left(c, \varepsilon_{11}\right) = E_0 + k\left|\varepsilon_{11}\right| c^2 \tag{6}$$

where c is a measure of the transverse compaction namely the local cross section area variation and ε_{11} is the longitudinal strain. The coefficient values k and E_0 have been identified by an inverse method from biaxial tension tests because these test lead to a significant compaction of the yarn [4]. It was shown in [4] that in order to have a continuum with a yarn type behaviour, i.e. null bending stiffness (or very low), shear moduli have to be null or very low. Poisson ratios are supposed to be null. The values of the material properties used for the glass plain weave of this study are listed in table 1.

Table1. Material parameters

Longitudinal Young modulus E1	35400 MPa
Transverse Young modulus	$0.2+8.10^4\left\|\varepsilon_{11}\right\|c^2$ MPa
Poisson ratios	0
Shear moduli	20 MPa

3 MESOSCOPIC SIMULATION OF IN-PLANE SHEAR OF WOVEN REINFORCEMENTS

This section addresses the way to perform mesoscopic scale simulations of the mechanical behavior of composite reinforcements. Here, the case of pure shear of fabrics is presented.

First the geometry of the model has to be defined. It must insure that there are neither unexpected penetrations nor spurious voids between the yarns. The mesh was realized based on these considerations [5].

Second the periodicity of the reinforcement is used to set the size of the geometric model. The smallest geometry to be modeled is that of a representative unit cell of the reinforcement, i.e. the smallest pattern permitting reconstructing the whole fabric by translations only. Its choice is not unique. Nevertheless considering the application of boundary conditions, the choice of a unit cell with material boundaries is preferred (see the deformed cell, fig. 2). In some particular cases like biaxial tension tests, the symmetry properties of the reinforcement and the test kinematics allow reducing the size of the model. This is not the case of pure shear because the symmetries of the geometry get lost due to the kinematics of shear. The periodicity of the reinforcement must be guaranteed during the test, which is achieved through appropriate boundary conditions. Due to periodicity, the displacement field is in the form:

$$\underline{\boldsymbol{\varphi}}(\underline{\mathbf{X}})=\underline{\boldsymbol{\varphi}_{\mathrm{m}}}(\underline{\mathbf{X}})+\underline{\mathbf{w}}(\underline{\mathbf{X}}) \tag{7}$$

with $\boldsymbol{\varphi}_{\mathrm{m}}$ denoting the macroscopic (known) displacement field, namely the pure shear field in this paper, and **w** the periodic (unknown) local displacement field. Considering two points of the boundary, **X** and **X'**, which are images of each other by virtue of periodicity, and using the periodicity condition **w**(**X**) = **w**(**X'**), one obtains the kinematical boundary conditions for the model:

$$\underline{\boldsymbol{\varphi}}(\underline{\mathbf{X}}')-\underline{\boldsymbol{\varphi}}(\underline{\mathbf{X}})=\underline{\boldsymbol{\varphi}_{\mathrm{m}}}(\underline{\mathbf{X}}')-\underline{\boldsymbol{\varphi}_{\mathrm{m}}}(\underline{\mathbf{X}}) \tag{8}$$

This equation consists of a set of relationships between the displacement degrees of freedom of each pair (**X**, **X'**) of the boundary.

4 RESULTS

The previous two sections lead to defining different aspects needed for a FE analysis of the plane sheared unit cell: geometry, mesh, boundary conditions, constitutive model of the fibrous medium. In this section, the results of simulations are shown and compared to experimental picture frame results. The first objective of the calculation of the deformed unit cell consists in numerically determining the shear behaviour curve similarly to the one obtained experimentally from a picture frame test (fig. 3). These experimental tests are tricky and cannot be performed during design or optimisation stages of a woven reinforcement. A virtual FE analysis is an interesting alternative. The second objective is the computation of the deformed geometry. This one can be used in damage prediction analyses or to compute the actual permeability of the woven reinforcement after it is draped on the final shape of the composite part. Permeability is numerically estimated by simulating the resin flow in the woven unit cell [6]. Significant variation of geometry during shear deeply modifies this permeability.

The curves of shear angle versus shear torque are plotted on fig. 3. The shear locking is properly captured and the agreement with the experimental curve is very good.

The deformed shape obtained from the analysis of in-plane shear is shown on fig. 2 for a shear angle of 53°. The local compaction is plotted and this value can locally reach 39%. Though this deformed geometry doesn't suffer any major distortion or defects, it can not be evaluated further at the moment. It is planed to use tomography as an observation tool of deformed woven reinforcements in order to compare the obtained images with the simulated shape.

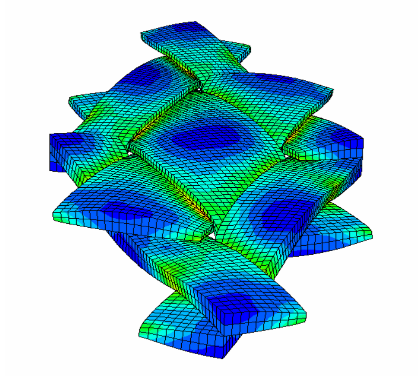

Fig. 2. Deformed unit cell. Plots of local compaction.

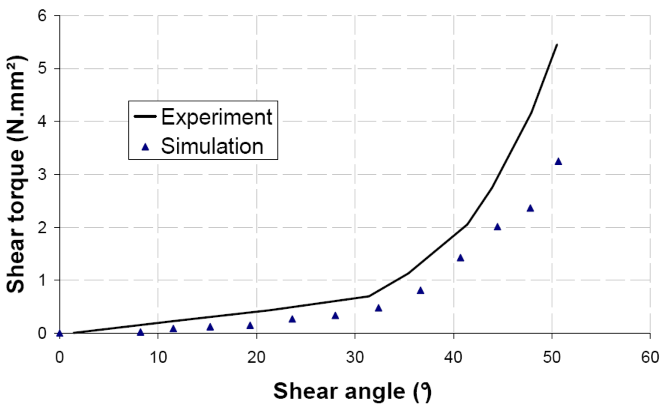

Fig. 3. Shear curve, simulation vs picture frame experiment.

5 CONCLUSIONS

A method for the analysis of mesoscopic deformation of woven reinforcements was presented. It involves two important aspects, the yarn constitutive model based on a specific hypo-elastic model and the periodic boundary conditions.

From these analyses, the mechanical behaviour of the reinforcement can be determined. It is used in several applications like finite element forming simulations or at the design stage of a fabric. The determination of the deformed geometry of the reinforcement is another important result since it is useful in fluid flow simulations aimed at computing the permeability. It can also be used for damage prediction simulation.

The results presented are good compared to the experiments. However, further investigations are needed to evaluate the deformed geometry. This is the point of the current work in progress using tomography as an observation tool.

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