Permanent strain and damage formulation in forming processes of filled-elastomer materials Mullin effect

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ABSTRACT: The rubbery environments present the same mechanical behaviour as hyperelastic or visco-hyperelastic material. Regarding the filled-elastomers, the high degree of deformability often generate microscopic and voluminal defects leading to the creation of micro-cracks inside the structure. It is important to develop sophisticated models to take into account the various physical phenomena and particularly the degradation of the matter by damage (or Mullins effect). The proposed approach is used to couple material degradation by damage with permanent strain in filled-elastomer materials. The efficiency of the method is validated with some examples.

KEYWORDS: Filled-Elastomers, Hyperelasticity, Damage, Tensile and Shear tests.

1 INTRODUCTION

Rubber-like materials (natural rubber, filled-elastomers and synthetic elastomers) are widely used in engineering applications (vehicle, shock absorbers, tyres, engine mounts). These materials are characterised by large deformations and exhibit a strongly non-linear behaviour [4]. When these rubber-like materials are subjected to cyclic loading, damage induced stress softening phenomena (Mullins effect) are observed.

This phenomenon is isotropic and depends only on the history of the material deformations. In the literature, the behaviour of rubber-like materials depends mostly on the strain-energy function [6] and the challenge is the development of a constitutive model for the Mullins effect with permanent deformation [5, 2]. This theory is based on the introduction of two internal variables into the strain-energy function which respectively characterize the stress softening and the appearance of a persistent deformation induced by a change of material properties (anisotropy) during loading-unloading tensile cycles. The expression of these two internal variables depends on the primary loading path from which all unloading path is initiated. Without loss of generality, the curve of the primary loading is associated to a virgin and isotropic material in which these variables are supposed inactive. This assumption limits the model to be applied to damaged filled-elastomers subjected to large deformation until rupture. A first work was done in [3] where we propose to enrich a non-linear coupled damaged hyperelastic behaviour of rubber-like material in order to reproduce both the Mullins effect in cyclic loading with a permanent effect and the degradation of the matter. In the framework of Continuum Damage Mechanics (CDM), a new variable is introduced in the strain-energy function in order to take into account the damage closely related to the material degradation and particularly to the primary load [1].

In addition, we propose in this study, a multiplicative decomposition of the gradient of deformation into an elastic and a damage part. The formulation is described and examples are given to show the efficiency of the approach.

2 CONSTITUTIVE MODELING

2.1 *Basic kinematics*

We consider a rubber-like solid regarded as a continuous body. To formulate a three-dimensional constitutive theory to represent the nonlinear behaviour of filled elastomers, the multiplicative decomposition of the deformation gradient into an elastic and a damage part is used :

$$\mathbb{F} = \mathbb{F}^e \cdot \mathbb{F}^d \tag{1}$$

where $\mathbb{F}^d$ is the fictitious intermediate configuration. In addition, the damage part of the gradient is assumed to be isochoric. This assumption is consistent with observations of volumetric constitutive behaviour of filled rubber. Finally, we deduce the elastic left Cauchy-Green tensor and its rate :

$$\mathbb{B}^e = \mathbb{F}^e \cdot (\mathbb{F}^e)^T = (\mathbb{V}^e)^2 \tag{2}$$

$$\dot{\mathbb{B}^e} = \mathbb{L} \cdot \mathbb{B}^e + \mathbb{B}^e \cdot \mathbb{L} - 2\mathbb{V}^e \cdot \mathbb{D}^{d*} \cdot \mathbb{V}^e \tag{3}$$

where $\mathbb{L} = \dot{\mathbb{F}}\mathbb{F}^{-1}$ is the rate tensor of deformation, $\mathbb{V}^e$ is the pure elastic strain tensor (i. e. $\mathbb{F}^e = \mathbb{V}^e.\mathbb{R}^e$) and $\mathbb{D}^{d*} = \mathbb{R}^e \cdot \mathbb{D}^d \cdot (\mathbb{R}^e)^T$ is the rate tensor of permanent deformation turned (i. e. $\mathbb{D}^d$ is the symmetric part of $\dot{\mathbb{F}^d}\mathbb{F}^{d^{-1}}$).

2.2 *Thermodynamics background*

In the theory of hyperelasticity we consider the free strain-energy function W which depends overall on the elastic motion $\mathbb{F}^e$, and the internal variables Ξ_i associated to physical phenomena inside the material. For simplification of calculations and without losing generalities, we will suppose an incompressible material, i.e. $J = J^e = 1$, and then, all motion tensors are equal to their distorsional parts. We will keep the same notation to express W with different mechanics motion elements :

$$W(\mathbb{F}^e, \Xi_i) = W(\mathbb{B}^e, \Xi_i) = W(\lambda_1^e, \lambda_2^e, \lambda_3^e, \Xi_i) \tag{4}$$

where λ_i^e (i $\in$ {1,2,3}) are the principal Eulerean stretches (i.e. the eigenvalues of $\mathbb{V}^e$). And in an isotherm framework, we write the inequality of Claudius-Duhem :

$$\mathbf{D} = \sigma : \mathbb{D} - \dot{W} \geq 0 \tag{5}$$

where σ is the Cauchy stress tensor.

2.3 *Hyperelastic model coupled to damage*

In this work, we propose to couple the Ogden form of strain-energy function with an internal state variable D, which take account the irreversible degradation of the matter following isotropic CDM Theory :

$$W(\lambda_i^e) = (1-D)\sum_{i=1}^{3}\frac{\mu_i}{\alpha_i}.\left((\lambda_1^e)^{\alpha_i} + (\lambda_2^e)^{\alpha_i} + (\lambda_3^e)^{\alpha_i} - 3\right) \tag{6}$$

D is a scalar which evolves from 0 (virgin part) and 1 (theoritical rupture).

2.4 *Damage evolution*

In order to describe the damage evolution, we consider W as a function expressed with $\mathbb{B}^e$, and to simplify we keep the same notation. From the inequality of Clausius-Duhem (5) and (3), we calculate :

$$\dot{W} = 2\left(\mathbb{B}^e\frac{\partial W}{\partial \mathbb{B}^e}\right)^{dev} : \mathbb{D} - 2\mathbb{V}^e\frac{\partial W}{\partial \mathbb{B}^e}\mathbb{V}^e : \mathbb{D}^{d*} + \frac{\partial W}{\partial D}\dot{D} \tag{7}$$

After some calculations, we obtain the following relations :

$$\begin{cases} \sigma = 2(1-D)\left(\mathbb{B}^e\frac{\partial W_0}{\partial \mathbb{B}^e}\right)^{dev} - p\mathbb{I} \\ \mathbf{D} = \mathbf{Y}\dot{D} + \mathbb{T}^* : \mathbb{D}^{d*} \geq 0 \end{cases} \tag{8}$$

where p is a Lagrange multiplier associated with the incompressibility assumption. $\mathbf{Y}$ and $\mathbb{T}^*$ are the thermodynamic forces respectivily associated to the damage D and the permanent rate tensor of deformation $\mathbb{D}^{d*}$, given by :

$$\mathbf{Y} = -\frac{\partial W}{\partial D} = W_0(\mathbb{B}^e) \tag{9}$$

$$\mathbb{T}^* = 2(1-D)\mathbb{V}^e\frac{\partial W_0}{\partial \mathbb{B}^e}\mathbb{V}^e \tag{10}$$

Following the framework of the Generalised Standard Materials (GSM) theory in the case of rate-independent flow, a damage criterion is given by :

$$f(\mathbf{Y}; D) = \mathbf{Y} - Q(D) \leq 0 \tag{11}$$

An unloading, neutral loading or loading from a damage state shall be added to the criterion (11) according to the standard time independent flow theory. $Q(D)$ being a differentiable positive and increasing function of D, representing the size of the damage surface in the $\mathbf{Y}$-space and is given by :

$$Q(D) = Q\sqrt[n]{D + D_0} \tag{12}$$

Q is the intensity expressed in term of energy, n is a parameter describing the linearity of the damage effect evolution, and D_0 is a positive threshold of non-damage effect (usually $D_0 << 1$). In the same space, a flow potential is introduced to complete a non-associative damage flow :

$$F(\mathbf{Y}, \mathbb{T}^*; D) = \mathbf{Y} + \frac{R}{2}\,\frac{\mathbb{T}^* : \mathbb{T}^*}{1 - D} \tag{13}$$

where R is a parameter describing the intensity of the permanent deformation. Following the standard normality argument the damage and the permanent rate tensor evolution (complementary laws) are given by :

$$\dot{D} = \begin{cases} \dot{\delta}\frac{\partial F}{\partial \mathbf{Y}} & \text{if} \quad f = 0 \\ 0 & \text{if} \quad f < 0 \end{cases} \tag{14}$$

$$\mathbb{D}^{d*} = \begin{cases} \dot{\delta}\frac{\partial F}{\partial \mathbb{T}^*} & \text{if} \quad f = 0 \\ 0 & \text{if} \quad f < 0 \end{cases} \tag{15}$$

where $\dot{\delta}$ is the damage multiplier given by the consistency condition $\dot{f} = 0$, i. e. :

$$\dot{\delta} = \frac{2\frac{\partial f}{\partial \mathbf{Y}}\left(\mathbb{B}^e \frac{\partial \mathbf{Y}}{\partial \mathbb{B}^e}\right)^{dev} : \mathbb{D}}{2\frac{\partial f}{\partial \mathbf{Y}}\mathbb{V}^e\frac{\partial \mathbf{Y}}{\partial \mathbb{B}^e}\mathbb{V}^e : \frac{\partial F}{\partial \mathbb{T}^*} - \frac{\partial f}{\partial D}\frac{\partial F}{\partial \mathbf{Y}}} \tag{16}$$

3 NUMERICAL APPLICATION

In order to validate our approach, three tests are proposed (uniaxial, equibiaxial and pure shear) to study the damage and the stress evolution in filled-elastomer material. The material parameters of the specimen made from a particle-reinforced compound with 60 phr of carbon black are given in Table 1 (see [2] for more information) :

Table 1: Material parameter

μ_1	-1.528380 MPa	
μ_2	0.222564 MPa	
μ_3	-1.13418.10^{-3} MPa	
α_1	-1.011467	Elasticity parameter
α_2	4.2047799	
α_3	-4.398598	
Q	15	
n	2	
D_0	10^{-6}	Damage parameter
R	0.01	

After some simplifications, the model is reduced to a scalar equation (damage criterion) and a tensoriel equation (evolution of $\mathbb{B}^e$) :

$$\begin{cases} W_0(\mathbb{B}^e) - Q\sqrt[n]{D + D_0} & = 0 \\ \dot{\mathbb{B}^e} - \mathbb{L}\mathbb{B}^e - \mathbb{B}^e\mathbb{L}^T + 4\,R\,\dot{D}\,\mathbb{B}^e\frac{\partial W_0}{\partial \mathbb{B}^e}\mathbb{B}^e & = 0 \end{cases} \tag{17}$$

The Theta-method time discretisation for flow variables and the Newton-Raphson method for non-linear equation resolution are used. The deformation gradient $\mathbb{F}$ is known at the beginning and at the end of the increment and all the other elements are known at the beginning of the increment. ΔD and $\Delta \mathbb{B}^e$ are the principal unknowns. When the system (17) is resolved, the stress can be calculate from the hyperelastic behaviour (8) with the new values of D and $\mathbb{B}^e$. The proposed model is implemented in ABAQUS Software using user's Subroutine (UMAT).

In the uniaxial and equibiaxial condition, the principal stretches are respectivily $\{\lambda,\ \lambda^{-1/2},\ \lambda^{-1/2}\}$ and $\{\lambda,\ \lambda,\ \lambda^{-2}\}$. According to the multiplicative decomposition of $\mathbb{F}$, the total stretch is decomposed into two parts : An elastic part (λ^e) and a damage part (λ^d), with the relation : $\lambda = \lambda^e\lambda^d$. The responses of the stress (σ) and the damage (D) versus elongations (λ and λ^e) are presented in figures 1 and 2 where the total stretch varies linearly until 400% of deformation (i. e. $\lambda_{max} = 5$).

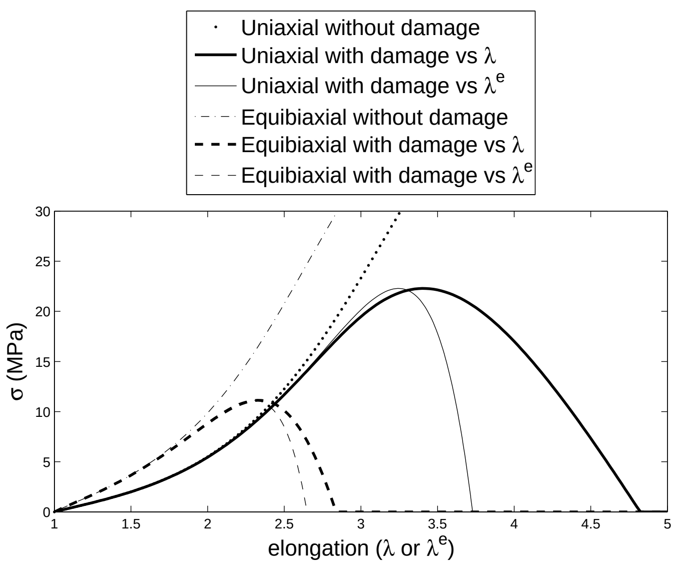

Figure 1: Stress in uniaxial and equibiaxial tests versus elongation.

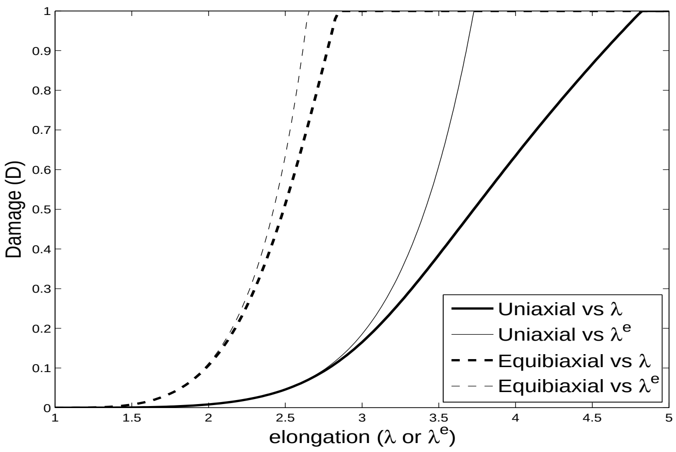

Figure 2: Damage in uniaxial and equibiaxial tests versus elongation.

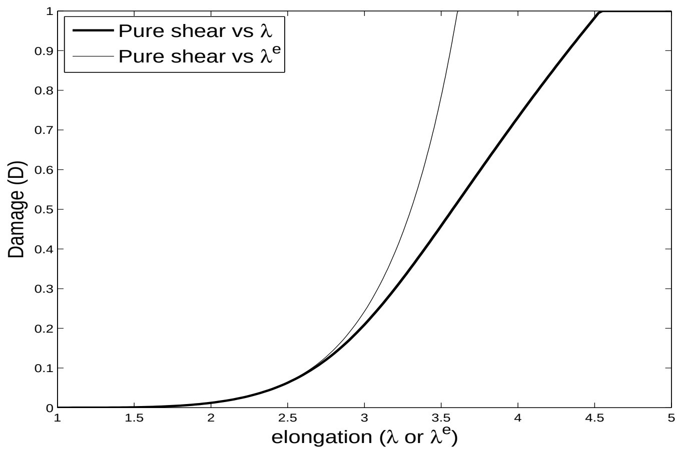

Figure 4: Damage in pure shear test versus elongation.

The comparison between with and without damage is presented in figure 1 and we can note that the damage is growing non-linearly (see figure 2) and affect the stress softening evolution during loading until rupture (ϵ_{rupt} equal to 420% in uniaxial and 190% in equibiaxial tensile test). Overall depending on λ^e, a difference between the two curves (vs λ^e and vs λ) is observed for each test due to the appearance of a permanent strain induced by damage.

Finally, we study the pure shear test which the principal stretches are $\{\lambda, 1, 1/\lambda\}$. In the same way the responses in stress and damage are shown in figures 3 and 4 and we observe the same physical variation.

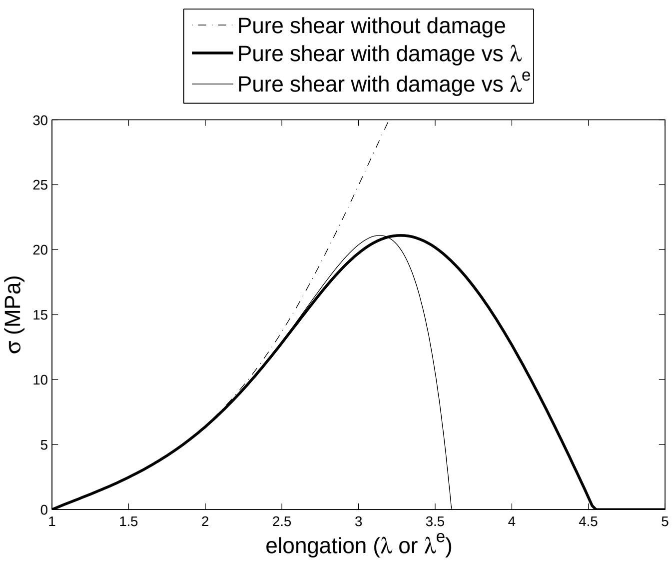

Figure 3: Stress in pure shear test versus elongation.

The response with and without damage is also compared. Damage affect stress softening until ϵ_{rupt} equal to 375% and still produce a permanent set.

4 CONCLUSION

In this study, we have formulated a rate-independent model that captures the effects of material degradation and permanent set for a class of filled-elastomers. In the framework of CDM, an irreversible state variable D model the isotropic damage effect associated to the creation of micro-cracks inside the structure. The validation of the proposed model is illustrated in uniaxial, equibiaxial and pure shear tests. Applications to filled-elastomer forming is in progress using the fully coupled model.

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