Tensile and Shear Deformation Modelling of Woven Fabrics

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1 INTRODUCTION

Shear is the main deformation mode during

ABSTRACT: This paper describes the application of elastica theory to modelling biaxial and shear deformation behaviour of woven fabrics. The normal force between the interlacing yarns has been computed and updated using the biaxial tensile model, since the friction couple at yarn intersections is a function of normal force.

Key words: Elastica, Biaxial Deformation, Shear Deformation, Material forming

composite forming process [1] such as thermo-forming. To study the behaviour of woven fabrics during shear it is important to predict the frictional resistance which depends on contact area, normal load due to yarn tensile load and surface properties. In this paper the theory of elastica is first developed to obtain the defining differential equations. These equations are then used to model the biaxial deformation behaviour of plain woven fabrics. In addition to load-elongation behaviour in the plane of the fabric, the important output is the normal load at cross-over points, in the relaxed and deformed fabric. The normal load determines the friction couple at yarn intersections and hence relates to the shear deformation. The elastic part of the shear deformation model, as described here, is the bending required for curvature change in the yarn paths during shearing.

2 PLANAR ELASTICA THEORY

Figure 1 shows a small element of an elastica and the forces and moments acting o it. From differential geometry:

$$\frac{dx}{ds} = w_x \qquad (1)$$

$$\frac{dy}{ds} = w_y \qquad (2)$$

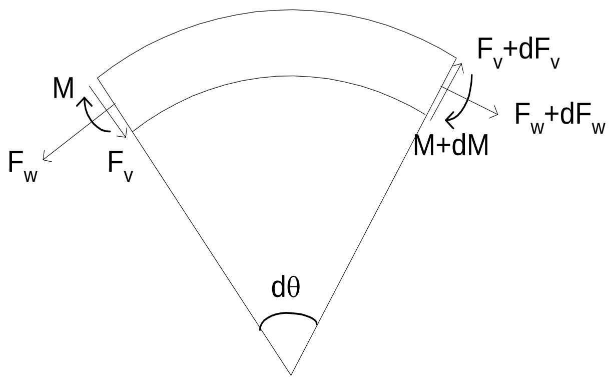

Figure 1: Small curved element of elastica

Nomenclature	
x, y	Global coordinates.
v, w	Local coordinates.
s	Distance measured along centre line of deformed elastica.
w_x, w_y	Direction cosines of local coordinate, *w*.
M	Moment on small element of elastica.
F_v	Internal force component, perpendicular to centreline of elastica.
F_w	Internal force component, tangential to centreline of elastica.
p	Curvature of centreline of elastica.
E_b	Bending rigidity of elastica.
d	Thickness or minor diameter of elastica.
a	Major diameter of elastica.
L	Distance between yarns in woven cloth.
T	Friction Couple
F_y	Normal force at yarn intersection.
μ	Coefficient of friction between yarns

$$\frac{dw_x}{ds} = -p.w_y \qquad (3)$$

$$\frac{dw_y}{ds} = p.w_x \qquad (4)$$

Taking moments about center of element in Figure 1 gives:

$$\frac{dM}{ds} = F_v \qquad (5)$$

Since $M = E_b.p$ equation (5) can be rewritten as follows:

$$\frac{dp}{ds} = \frac{F_v}{E_b} \qquad (6)$$

Due to equilibrium of forces on the element in Figure 1:

$$\frac{dF_v}{ds} = p.F_w \qquad (7)$$

$$\frac{dF_w}{ds} = -p.F_v \qquad (8)$$

The elastica solution results from the numerical integration, such as Runge-Kutta, of the series of first order differential equations. The integration begins at one end of the elastica, where all the initial values must be specified, and proceeds to the other end, where the boundary values are obtained. In practice a certain number of initial values will be unknown. The problem may be solved numerically by the Newton-Raphson technique provided sufficient boundary values are given.

3 BIAXIAL MODEL OF WOVEN FABRIC

Both the force-equilibrium [2] and the energy methods [3, 4] have been extensively used to model the tensile behaviour of woven fabrics. In this paper the force-equilibrium approach is adopted. It is assumed that the warp and weft yarns behave like extensible elasticas that interact with each other. The yarn cross-section is assumed to be lenticular with the major and minor diameters given. Table 1 below lists the initial values that are known and unknown for the warp & weft. The subscripts 1 and 2 refer to warp and weft respectively. The parameter D is the sum of the minor diameters of the warp & weft. External forces Fx_1 and Fx_2 are given. The forces Fy_1 and Fy_2 act in a direction normal to the fabric plane at the crossover points and are unknown. In the case of plain woven fabrics $Fy_1 = Fy_2$.

The problem of plain woven fabric is basically that of a planar elastica with 3 unknown initial values. A unique solution of the model will require, therefore, the knowledge of three boundary values. These are:

- $p_1 = 0$ at $s = L_1/2$ where $L_1/2$ is half the free length of warp between consecutive weft.
- $p_2 = 0$ at $s = L_2/2$ where $L_2/2$ is half the free length of weft between consecutive warp.
- $y_1 + y_2 = \frac{d_1 + d_2}{2}$ (9)
  at $s = L_1/2$ for warp and $s = L_2/2$ for weft.

Table 1: Initial conditions

Initial Conditions at s=0	
Warp	Weft
$x_1=0$; $y_1=0$; $p_1=2/D$ or unknown; $wx_1=0$ or unknown; $wy_1 = \sqrt{(1 - wx_1^2)}$; Fx_1: Load applied along x-axis on warp Fy_1: Load applied along y-axis on warp	$x_2=0$; $y_2=0$; $p_2=2/D$ or unknown; $wx_2=0$ or unknown; $wy_2 = \sqrt{(1 - wx_2^2)}$; Fx_2: Load applied along x-axis on weft Fy_2: Load applied along y-axis on weft

In this work, the biaxial model has been used to predict the inter-yarn normal forces resulting form the thread-line tensions applied during bias extension (figure 2).

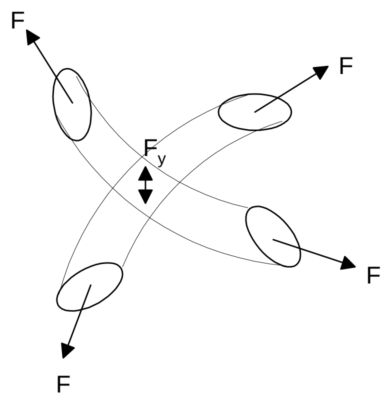

Figure 2: Inter-yarn force F_y due to yarn tension F

4 SHEAR MODEL

4.1 Initial Shear Stiffness

Grosberg and Park [5] produced a mathematical analysis for predicting shear behaviour of woven fabrics. The shear model developed here is based on the Grosberg approach [5] as well as that of Skelton [6]. Several authors have contributed to shear modelling more recently [7,8]. It is worth pointing out that Kawabata [9] used a very different approach where experiments are used to determine the coefficients of the shear force and deformation relationship. At low levels of shear stress there is not enough moment available to move the yarns in the fabric at the interaction regions and they behave as if they were welded junctions. In this case deformation takes place in the free length between crossover regions and the deformation is essentially that of cantilever. The set of differential equations developed above for plane elastica are integrated to obtain the moment at yarn crossover point given the deformation. Again the solution is obtained through solving a boundary value problem with one unknown initial value, the curvature at the 'welded junction' and with zero curvature, halfway along the free yarn length, as boundary value. The localised shear force, within a unit cell, is then given as follows:

$$F_s = \frac{p.E_b}{L} \qquad (10)$$

4.2 Shear Deformation Beyond Initial Region

As soon as the stresses are large enough to overcome the frictional restraints that are acting at the crossover regions the system start to slip. The shear stiffness then remains fairly constant until transverse compaction of the yarns causes a sharp increase in stiffness. The mechanism of fabric shear is quite complicated. Once the yarns start to slip, the friction couple increases with shear load due to increase in normal loads at the intersections. Also the yarn paths, hence curvatures, in the fabric are changing with shear deformation, hence further bending is required. Hence the external moment will equal the sum of the frictional couple and the moment required for the curvature change of the yarns in the fabric. If it is assumed that the contact area is circular with diameter equal to the major diameter, *a*, of the yarn, then friction couple is:

$$T = \mu \frac{4F_y}{\pi a^2} \int_0^{a/2} \frac{2\pi a^2}{4} dr = \frac{1}{3} \mu F_y a \qquad (11)$$

The normal load depends on the axial load in the yarn and is obtained from the biaxial model of the woven fabric, described above. The axial load for shear load *F*, shown in Figure 3a is $F\cos\theta$ where θ is half the angle between warp and weft. The moment, M, due to change in yarn path curvature is given by [6]:

$$M = E_b \sin^2(45-\theta).p \qquad (12)$$

where p is the curvature of the yarn path, at intersections, in the orthogonal or undeformed fabric. The relationship between shear load and deformation is therefore:

$$F\sin\theta = E_b \sin^2(45-\theta).p + \frac{1}{3}\mu F_y a \qquad (13)$$

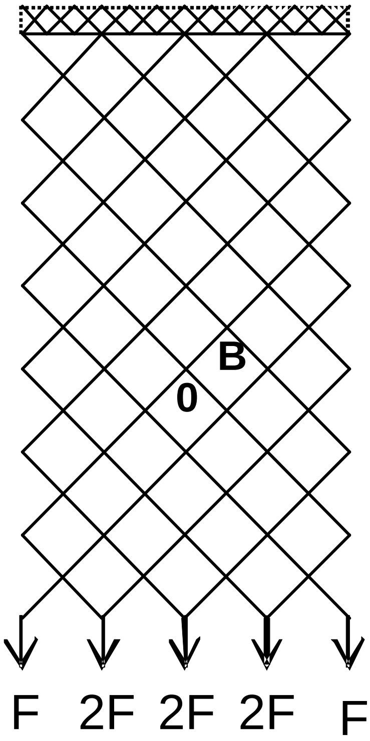

Figure 3a: Bias Extension

Figure 3 shows the bias extension of woven fabric and Figure 4 illustrates the forces on a unit cell taken from Figure 3.

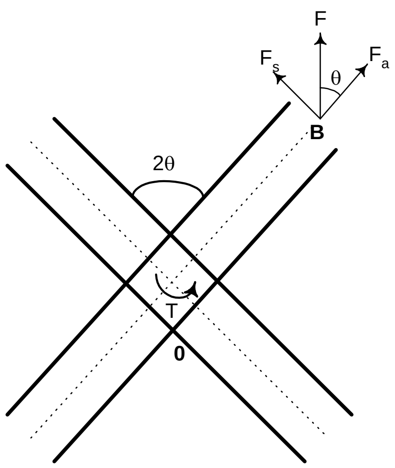

Figure 4: Deformed Unit Cell

Figure 5 shows a comparison of the simulation results with bias extension test results obtained for a plain woven glass fibre fabric with yarn bending rigidity of 0.6 Nmm^2. The specifications of the fabric are in Table 2:

Table 2: Material Specifications

Fabric	**Yarn**	**Filament**
Structure: Plain Weave	Linear density: 1200 Tex	Type: E Glass
Ends/cm: 2.4	Number of filaments: 2000	Modulus: 72.4 GPa
Picks/cm: 2.4	Flexural Rigidity: 0.6 Nmm^2	Density: 2.5g/cm^3

The graph indicates that the general behaviour of shear deformation is achieved by the model. There is a good agreement between the model and the experimental graphs in the initial elastic region. Subsequent stiffening due to yarn transverse compaction and the increase in yarn contact area have not been considered in this model.

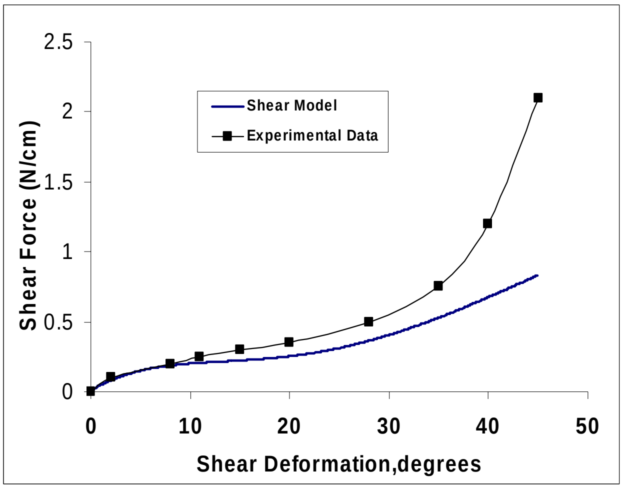

Figure 5: Simulation Result of Shear Model

5 CONCLUSIONS

A woven fabric biaxial model based on large deformation of elastica has been described. This model has been used to obtain values for normal loads at yarn intersections which in turn determine the friction couple during shear deformation. Further the elastica theory has been used to model the initial stiffness region when external moment is not sufficient to overcome frictional restraint at crossover regions. Further work needs to be carried out to account for changing contact area and to model large shear deformation as then yarn transverse flattening should also be considered.

REFERENCES

1. Long A.C, Rudd C.D, Blagdon M, Smith P. Characterizing the processing and performance of aligned reinforcements during preform manufacture. Composites: Part A 1996; 27A:247-53
2. P. Grosberg and S. Kedia, The mechanical properties of woven fabrics, Part 1: The initial load extension modulus of woven fabrics, *Text. Res. J.*, 36, 76, 1966.
3. J.W.S. Hearle and W.J. Shanahan, An energy method for calculations in fabric mechanics, Part 1: Principles & Methods, *J. Text. Inst.*, 69,4, 1978.
4. J.W.S Hearle and W.J. Shanahan, An energy method for calculations in fabric mechanics, Part II: Examples of applications of the method to woven fabrics, *J. Text. Inst.*, 69, 4, 1978.
5. P. Grosberg and B.J Park, The mechanical properties of woven fabrics, part V: The initial modulus and the frictional restraint in shearing of plain weave fabrics. Text. Res. J. 1966:420-31.
6. J. Skelton, Shear of woven fabrics, Proceedings of the NATO Advanced Study Institute on Mechanics of Flexible Fibre Assemblies, SIJTHOOF & NOORDHOOF 1980, Netherlands.
7. J. Page and J. Wang, Prediction of shear force and an analysis of yarn slippage for a plain-weave carbon fabric in a bias extension state. Composites Science and Technology 60, 2000, 977-986.
8. S.V.Lomov and I. Verpoest, Model of shear of woven fabric and parametric description of shear resistance of glass reinforcements, Composites Science and Technology 66, 2006, 919-933.
9. Kawabata S., Niwa M., and Kawai H. 'The Finite Deformation Theory of Plain Weave Fabrics Part III: The Shear Deformation Theory', J. Textile Inst., 64 (1), (1973) 62-85.