A Local Multi-grid X-FEM approach for 3D fatigue crack growth

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ABSTRACT: One proposes a local multi-grid X-FEM method for 3-D crack propagation simulation in large structures. Enrichments allow the asymptotic behaviour on the crack front to be well described and the structure doesn't need to be remeshed during the crack propagation. However, the different scales involved in fracture mechanics problems can differ from several orders of magnitude and industrial meshes are usually not designed to take into account small cracks. Enrichments are therefore useless if the crack is too small compared to the element size. The X-FEM is thus coupled with a local multi-grid algorithm. The link between the scale of the structure and the scale of the crack is hence possible. Moreover, multi-grid techniques are very efficient in term of cpu time. The coupling of X-FEM with the multi-grid algorithm is then described and specific computational aspects that deal with intergrid operators, the multi-scale enrichment strategy and level sets are pointed out. In particular, a specific enrichment strategy, based on a multi-grid convergence rate criterion is proposed. Furthermore, the use of a single independent structured mesh for the level sets description is shown to be very robust and accurate. Finally, examples that illustrate the good accuracy and efficiency of the method are given and especially in the case of 3D fatigue crack propagation.

Key words: 3D crack growth, fatigue, local multi-grid, X-FEM, level sets

1 INTRODUCTION

The possibility to use meshes that are not conform to the crack as well as the introduction of singular components in the discrete field interpolation makes the extended finite element method a privileged tool for crack growth simulation [2, 18]. Furthermore, the X-FEM and level set combination allows to perform representations of 3D cracks with good accuracy and efficiency [3, 5, 20]. In the industry, structures are often meshed from models without accounting for defects such as cracks or inclusions. One of the crack growth mechanism is fatigue due to cyclic loading. The aim is to be able to prevent the structure from fracture and to analyse fatigue crack growth. Just after initiation, the scale of the crack can differ from several orders of magnitude from the scale of the structure. In this respect, many authors have proposed multi-scale methods which are able to couple such phenomena [8, 9, 11, 12]. In this presentation, he first step was the implementation of a global multi-grid algorithm within the X-FEM framework and was presented in a previous paper [1]. This work emphasized the high efficiency in cpu time but highlighted that mesh refinement is often required on localized areas only (cracks, inclusions, steep gradient zones). The link between the successive meshes is performed by intergrid operators (prolongation and restriction operators) built by means of the finite element shape functions. In the context of the extended finite element method, a special care in their construction is required since the displacement interpolation is mesh dependent, regarding the nature of the enrichment function on each node [1]. Both two and three dimensional examples are given to demonstrate the robustness and the efficiency of this combined LMG-X-FEM procedure.

2 A LOCAL MULTI-GRID X-FEM APPROACH

2.1 Intergrid operators with enrichments

Multi-grid techniques exploit the fact that high frequency components are computed on the fine mesh by iterative solvers in a very efficient way [13, 15]. However, computing the smooth components on this fine mesh would be very expensive. As a consequence, the displacement field is transferred on a coarser mesh where iterative solvers become efficient again. Solution components are computed on meshes where the solver has its best efficiency. In this presentation, a local multi-grid algorithm is proposed in order to couple in an optimal way the scale of the structure and the crack. Indeed, in the case of small cracks within large structures, the order of magnitude between the different scales ranges from several decades.

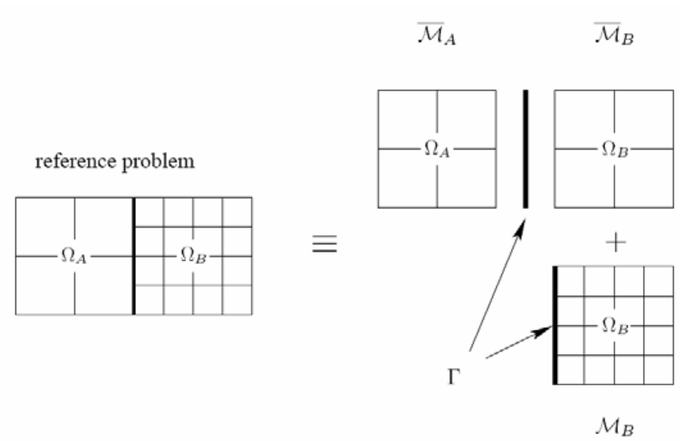

Fig. 1. A two level local multi-grid discretization

It is thus obvious to employ several levels to take the best advantage of its properties. Indeed, as shown in Reference [14], a ratio of 2 between two successive hierarchical grids seems to be optimal. Instead of being exactly solved, the coarse problem is approximated by performing recursive calls to the multi-grid procedure, using a still coarser discretization. The difficulty in defining the prolongation operator within the LMG-X-FEM framework comes from the possible difference in the shape functions basis used in two successive levels. The extended finite element method [2, 3] proposes to locally enrich the finite element interpolation. It has been shown in Reference [1] that if the displacement expression on the coarse space discretization handles the shape functions of the fine mesh, a perfect interpolation is achieved by performing a separate interpolation of the degrees of freedom, with respect to their enrichment type. In this case, the enrichments are said to be compatible. On the opposite case, the enrichments are said to be incompatible if there is no unique way to define the dofs of a node n from the coarse mesh to the fine one. Perfect interpolation is not possible on the area concerned by these incompatible enrichments since the fine shape functions basis is not included in the coarse one. An interpolation method has however been proposed in Reference [1]. Because this area is localized on a reduced set of elements, the error thus introduced contents only high frequency components which are easily removed in the relaxation step.

2.2 Optimal enrichment strategy

In the present subsection, the influence of different multilevel enrichment strategies on the LMG-XFEM convergence is studied. The aim is to define if all mesh levels have to be enriched. If the crack length is very small compared to the coarse levels element size, enrichments are not adapted to model the problem and they can lead to ill-conditioned stiffness matrices (enhancement of enrichment functions has been proposed in Reference [10] to handle small straight cracks in a single level two-dimensional formulation). In the present approach, the solution computed on the coarsest levels on refined areas do not have any physical sense and may not be enriched.

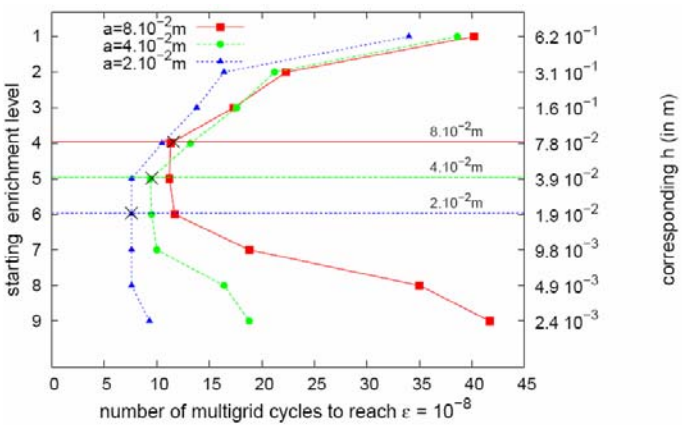

Fig. 2. Link between the finest enriched level and the rate of convergence for a given crack length

For this study, a square domain with a very small crack submitted to pure mode I is considered. The enrichment scheme is parameterized by the starting enrichment level. The enrichments may be performed from the coarsest level (level 1), from the finest one (level 9) or from any intermediate level. The number of multi-grid cycles necessary to reach a relative error of 10−8 versus the starting enrichment level ranging from 1 to 9 is considered for 3 crack lengths (see Figure 2). When the starting enrichment level is too fine (*ie.* only the finest levels are enriched), the problems described on the intermediate levels are too far from the physical model to help the algorithm to converge. However, if the starting enrichment level is too coarse (*ie.* almost all levels are enriched), a bad behaviour is also observed. This points out that an optimal convergence rate is achieved when the characteristic element size of the starting enrichment level is of the order of magnitude of the crack size.

3 AN INDEPENDENT MODELING OF THE CRACK SHAPE

The next point to be addressed is the definition of the level sets within the LMG-X-FEM framework. The use of level sets as an implicit representation of cracks is now a well established technique [3, 4, 5]. The iso-zero surface of a first signed distance function is used to describe the crack surface. The front is localized at the intersection of this unbounded surface with the iso-zero of a second level set field. Usually, these two fields are discretized on the mesh of the structure. For a n levels multi-grid strategy, this would involve to store n level set field pairs. Moreover, this would imply propagating independently these n level sets, which may lead to different crack representations afterward. The use of a new single structured independent mesh only devoted to the representation of both level sets is preferred to avoid these drawbacks. This new auxiliary mesh is only concerned with the level sets propagation. It further involves inherent advantages. (i) A better control of the crack description accuracy independently of the structure discretization is achieved (ie. the level set discretization can be finer than the mesh of the structure). (ii) Very robust schemes defined in the finite difference framework for the level sets operations defined by Sethian in [5] can be used. (iii) The level set mesh is localized in the area of interest and no more over the whole structure.

4 EXAMPLE

To illustrate the good accuracy and efficiency of the method, a 3D fatigue crack growth example is presented. The initial crack is a half circle of radius 0.2 mm. A 600 MPa tensile loading and a 50MPa shear loading is applied so that the crack is submitted to mixed mode. The material is an aluminum alloy with a 80 GPa Young's modulus and a 0.3 Poisson's ratio. The initial mesh is composed of 0.25 mm×0.25 mm×0.25 mm hexaedral elements and is obviously too coarse to take into account the crack despite the use of singular enrichments. The discretization is thus refined by adding three localized grid patches (see Figure 3). The patches dimensions are defined manually. One has to ensure that their size is adapted to correctly model the problem. Since the nodal spacing on the coarsest mesh is on the order of the crack radius and following the rule defined is section 3, all four levels are enriched. The level sets are supported by a fifth independent mesh. A good choice for the level sets discretization is to consider element sizes of the order of the finest ones used to model the structure.

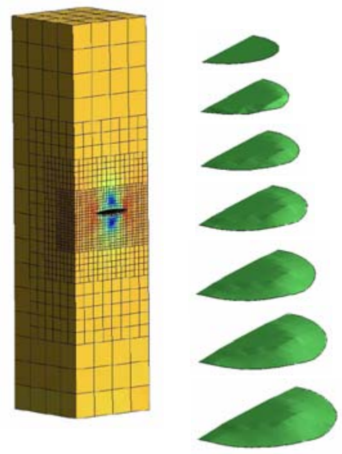

Fig. 3. Example of a 3D crack growth in mixed mode

This mesh does not cover the whole structure domain and is limited to the supposed crack propagation path. The only constraint is to consider a level set domain that covers the whole enriched zone of all levels. The crack extension is then computed by means of a three-dimensional Paris law and stress intensity factors by interaction integrals [17, 19, 20]. Figure 3 shows the crack and related surfaces at initial time and after seven propagation steps. In this example, one considers structured meshes, but this is not mandatory for the multi-grid algorithm.

5 CONCLUSIONS

The LMG-X-FEM method is able to account for a multi-scale problem ranging from the scale of the initial pre-existing crack to the scale of the structure itself. This method combines the extended finite element method, which avoids remeshing the structure during the crack propagation, with a local multi-grid technique. The discretization is locally refined in the crack area by adding local finer mesh patches. The proposed algorithm intrinsically handles incompatible patch interfaces. This is of great interest, particularly for three-dimensional problems or when the scale factors are important. Moreover, multi-grid techniques are very efficient in term of cpu time. Coupling this technique with X-FEM involves several specificities and developments. First of all, the intergrid operators have to account for the nature of enrichments on the different grids. An optimal enrichment strategy, based on a multi-grid convergence rate criterion has also been proposed. Finally, the use of the finite difference method as well as a single independent mesh for the level sets propagation has been shown to be very robust and accurate. Examples illustrate the accuracy and the efficiency of the proposed method, especially in the case of non-planar 3-D crack propagation. Note that the refined domain is defined by the user at the beginning of the simulation. An automatic mesh refinement algorithm such as the one proposed by Cavin *et al.* in [21] can also be integrated in the present method, as well as extensions to nonlinear behaviours [6, 7].

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